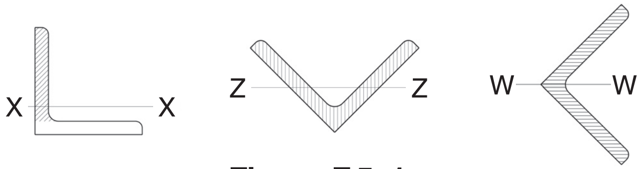

Figure F.5. 1

LIMIT STATE	M_n	b/t
yielding	$1.5F_{cy}S_c$	$b/t \le \lambda_1$
inelastic buckling	$\left[B_{br} - 4.0D_{br}(b/t)\right]S_c$	$\lambda_1 < b/t < \lambda_2$
elastic buckling	$\dfrac{\pi^2 ES_c}{(4.0b/t)^2}$	$b/t \ge \lambda_2$

where

$$\lambda_1 = \frac{B_{br} - 1.5F_{cy}}{4.0D_{br}}$$

$$\lambda_2 = \frac{C_{br}}{4.0}$$

Buckling constants B_{br}, D_{br}, and C_{br} are given in Tables B.4.1and B.4.2.

(2) If a leg is in uniform compression (Figure F.5.2):

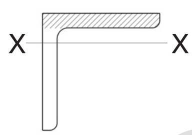

Figure F.5. 2

LIMIT STATE	M_n	b/t
yielding	$F_{cy}S_c$	$b/t \le \lambda_1$
inelastic buckling	$\left[B_p - 5.0D_p(b/t)\right]S_c$	$\lambda_1 < b/t < \lambda_2$
elastic buckling	$\dfrac{\pi^2 ES_c}{(5.0b/t)^2}$	$b/t \ge \lambda_2$

where

$$\lambda_1 = \frac{B_p - F_{cy}}{5.0D_p}$$

$$\lambda_2 = \frac{C_p}{5.0}$$

Buckling constants B_p, D_p, and C_p are given in Tables B.4.1and B.4.2.

b) For the limit state of yielding (Figure F.5.3):

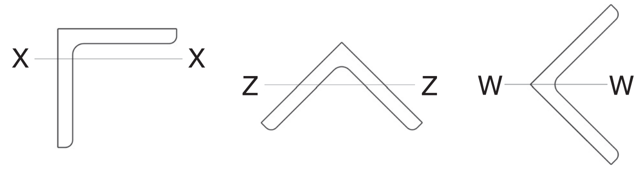

Figure F.5. 3

$$M_n = 1.5M_y \qquad \text{(F.5-1)}$$

where M_y = yield moment about the axis of bending.

c) For the limit state of lateral-torsional buckling:

(1) for $M_e \le M_y$, $M_n = (0.92 - 0.17M_e/M_y)M_e$ (F.5-2)

(2) for $M_e > M_y$, $M_n = (1.92 - 1.17\sqrt{M_y/M_e})M_y \le 1.3M_y$ (F.5-3)

where M_e = elastic lateral-torsional buckling moment from Section F.5.1 or F.5.2.

C_b between brace points shall be determined using Equation F.4-2 but shall not exceed 1.5.

F.5.1 Bending About Geometric Axes

Bending about a geometric axis is shown in Figure F.5.4. For combined axial compression and bending, resolve moments about principal axes and use Section F.5.2.

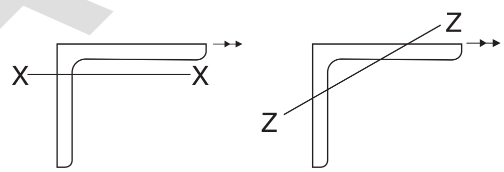

Figure F.5. 4

a) Angles with continuous lateral-torsional restraint: M_n is the lesser of:

(1) local buckling strength determined by Section F.5a.
(2) yield strength determined by Section F.5b.

b) Equal leg angles with lateral-torsional restraint only at the point of maximum moment: Strengths shall be calculated with S_c being the geometric section modulus. M_n is the least of:

(1) local buckling strength determined by Section F.5a.
(2) yield strength determined by Section F.5b.
(3) If the leg tip is in compression, lateral-torsional buckling strength determined by Section F.3c with

$$M_e = \frac{0.82Eb^4tC_b}{L_b^2}\left[\sqrt{1 + 0.78(L_bt/b^2)^2} - 1\right] \qquad \text{(F.5-4)}$$

If the leg tip is in tension, lateral-torsional buckling strength determined by Section F.3c with

$$M_e = \frac{0.82Eb^4tC_b}{L_b^2}\left[\sqrt{1 + 0.78(L_bt/b^2)^2} + 1\right] \qquad \text{(F.5-5)}$$

where
A_n = net area of the pipe or tube
A_{wz} = weld-affected area of the pipe or tube

For the limit states of shear yielding and shear buckling V_n is as defined in Section G.1 with

$$A_v = \pi(D_o^2 - D_i^2)/8 \quad \text{(G.4-3)}$$

where
D_o = outside diameter of the pipe or tube
D_i = inside diameter of the pipe or tube

and F_s determined from:

LIMIT STATE	F_s	λ
yielding	F_{sy}	$\lambda \le \lambda_1$
inelastic buckling	$1.3B_s - 1.63D_s\,\lambda$	$\lambda_1 < \lambda < \lambda_2$
elastic buckling	$\dfrac{1.3\pi^2 E}{(1.25\lambda)^2}$	$\lambda \ge \lambda_2$

where

$$\lambda_1 = \frac{1.3B_s - F_{sy}}{1.63D_s}$$

$$\lambda_2 = \frac{C_s}{1.25}$$

$$\lambda = 2.9\left(\frac{R_b}{t}\right)^{5/8}\left(\frac{L_v}{R_b}\right)^{1/4} \quad \text{(G.4-4)}$$

R_b = mid-thickness radius of a pipe or round tube or maximum mid-thickness radius of an oval tube
t = wall thickness
L_v = length of pipe or tube from maximum to zero shear force

G.5 RODS

The nominal shear strength V_n of rods is

For the limit state of shear rupture

For unwelded members

$$V_n = F_{su} A_n / k_t \quad \text{(G.5-1)}$$

For welded members

$$V_n = F_{su}(A_n - A_{wz})/k_t + F_{suw} A_{wz} \quad \text{(G.5-2)}$$

where
A_n = net area of the rod
A_{wz} = weld-affected area of the rod

For the limit state of shear yielding, V_n is as defined in Section G.1 with

$$A_v = \pi D^2/4 \quad \text{(G.5-3)}$$

where
D = diameter of the rod
$F_s = F_{sy}$ (G.5-4)

Appendix 6 Design of Braces for Columns and Beams

This appendix addresses strength and stiffness requirements for braces for columns and beams.

6.1 GENERAL PROVISIONS

The available strength and stiffness of bracing members and connections shall equal or exceed the required strength and stiffness, respectively, given in this appendix.

Columns with end and intermediate braced points that meet the requirements of Section 6.2 shall be designed using an unbraced length L equal to the distance between the braced points with an effective length factor $k = 1.0$. Beams with intermediate braced points that meet the requirements of Section 6.3 shall be designed using an unbraced length L_b equal to the distance between the braced points.

As an alternate to the requirements of Sections 6.2 and 6.3, a second-order analysis that includes initial out-of-straightness of the member to be braced shall be used to obtain the brace strength and stiffness requirements.

For all braces, $\phi = 0.75$ (LRFD), and $\Omega = 2.00$ (ASD), except that for nodal torsional bracing of beams, $\Omega = 3.00$.

6.1.1 Bracing Types

a) A relative brace controls movement of the braced point with respect to adjacent braced points.

b) A nodal brace controls movement of the braced point without direct interaction with adjacent braced points.

c) Continuous bracing is bracing attached along the entire member length.

6.1.2 Bracing Orientation

The brace strength (force or moment) and stiffness (force per unit displacement or moment per unit rotation) requirements given in this appendix are perpendicular to the member braced. The available brace strength and stiffness perpendicular to the member braced for inclined braces shall be adjusted for the angle of inclination. The determination of brace stiffness shall include the effects of member properties and connections.

6.2 COLUMN BRACING

6.2.1 Relative Bracing

The required strength is

$$P_{rb} = 0.004P_r \qquad (6\text{-}1)$$

The required stiffness is

$$\beta_{br} = \frac{1}{\phi}\left(\frac{2P_r}{L_b}\right) \qquad \text{(LRFD) (6-2)}$$

$$\beta_{br} = \Omega\left(\frac{2P_r}{L_b}\right) \qquad \text{(ASD) (6-2)}$$

where

L_b = distance between braces

For LRFD, P_r = required axial compressive strength using LRFD load combinations.

For ASD, P_r = required axial compressive strength using ASD load combinations.

6.2.2 Nodal Bracing

For nodal braces equally spaced along the column:
The required strength is

$$P_{rb} = 0.01P_r \qquad (6\text{-}3)$$

The required stiffness is

$$\beta_{br} = \frac{1}{\phi}\left(\frac{8P_r}{L_b}\right) \qquad \text{(LRFD) (6-4)}$$

$$\beta_{br} = \Omega\left(\frac{8P_r}{L_b}\right) \qquad \text{(ASD) (6-4)}$$

where

L_b = distance between braces. In Equation 6-4, L_b need not be taken less than the maximum unbraced length kL permitted for the column based on the required axial strength P_r.

P_r = required axial compressive strength

6.3 BEAM BRACING

Beams and trusses shall be restrained against rotation about their longitudinal axis at support points. Beam bracing shall prevent relative displacement of the top and bottom flanges (twist of the section). Lateral stability of beams shall be provided by lateral bracing, torsional bracing, or a combination of the two. Inflection points shall not be considered braced points unless they are provided with braces meeting the requirements of this appendix.

6.3.1 Lateral Bracing

Lateral braces shall be attached at or near the compression flange, except:

a) At the free end of cantilever members, lateral braces shall be attached at or near the tension flange.

b) For beams subjected to double curvature bending, lateral bracing shall be attached to both flanges at the brace point nearest the inflection point.

6.3.1.1 Relative Bracing

The required strength is

$$P_{rb} = 0.008M_r\, C_d\, /h_o \tag{6-5}$$

The required stiffness is

$$\beta_{br} = \frac{1}{\phi}\left(\frac{4M_rC_d}{L_bh_o}\right) \quad \text{(LRFD) (6-6)}$$

$$\beta_{br} = \Omega\left(\frac{4M_rC_d}{L_bh_o}\right) \quad \text{(ASD) (6-6)}$$

where

h_o = distance between flange centroids
C_d = 1.0 except $C_d = 2.0$ for the brace closest to the inflection point in a beam subject to double curvature
L_b = distance between braces
M_r = required flexural strength

6.3.1.2 Nodal Bracing

The required strength is

$$P_{rb} = 0.02M_r\, C_d\, /h_o \tag{6-7}$$

The required stiffness is

$$\beta_{br} = \frac{1}{\phi}\left(\frac{10M_rC_d}{L_bh_o}\right) \quad \text{(LRFD) (6-8)}$$

$$\beta_{br} = \Omega\left(\frac{10M_rC_d}{L_bh_o}\right) \quad \text{(ASD) (6-8)}$$

where

h_o = distance between flange centroids
C_d = 1.0 except $C_d = 2.0$ for the brace closest to the inflection point in a beam subject to double curvature
L_b = distance between braces. In Equation 6-8, L_b need not be taken less than the maximum unbraced length permitted for the beam based on the required flexural strength M_r.
M_r = required flexural strength

6.3.2 Torsional Bracing

Bracing shall be attached to the braced member at any cross section location on the member and need not be attached near the compression flange.

6.3.2.1 Nodal Bracing

The required strength is

$$M_{rb} = \frac{0.024M_rL}{nC_bL_b} \tag{6-9}$$

The required stiffness of the brace is

$$\beta_{Tb} = \frac{\beta_T}{\left(1-\dfrac{\beta_T}{\beta_{sec}}\right)} \tag{6-10}$$

If $\beta_{sec} < \beta_T$, torsional beam bracing shall not be used.

$$\beta_T = \frac{1}{\phi}\left(\frac{2.4LM_r^2}{nEI_yC_b^2}\right) \quad \text{(LRFD) (6-11)}$$

$$\beta_T = \Omega\left(\frac{2.4LM_r^2}{nEI_yC_b^2}\right) \quad \text{(ASD) (6-11)}$$

$$\beta_{sec} = \frac{3.3E}{h_o}\left(\frac{1.5h_ot_w^3}{12} + \frac{t_sb_s^3}{12}\right) \tag{6-12}$$

where

L = span length. In Equation 6-9, L_b need not be taken less than the maximum unbraced length permitted for the beam based on the required flexural strength M_r.
n = number of nodal braced points in the span
I_y = out-of-plane moment of inertia
C_b = beam coefficient determined in accordance with Section F.1.1
t_w = beam web thickness
t_s = beam web stiffener thickness
b_s = stiffener width for one-sided stiffeners (use twice the individual width for pairs of stiffeners)
β_T = overall brace system stiffness
β_{sec} = web distortional stiffness, including the effect of web transverse stiffeners, if any
M_r = required flexural strength

Web stiffeners shall extend the full depth of the braced member and shall be attached to the flange if the torsional brace is also attached to the flange. Alternatively, the stiffener may end a distance of $4t_w$ from any beam flange that is not directly attached to the torsional brace.

6.3.2.2 Continuous Bracing

For continuous bracing, use Equations 6-9 and 6-10 with the following modifications:

a) $L/n = 1.0$;
b) L_b shall be taken as the maximum unbraced length permitted for the beam based on the required flexural strength M_r;
c) The web distortional stiffness shall be taken as:

$$\beta_{sec} = \frac{3.3Et_w^3}{12h_o} \tag{6-13}$$

Example 6
PLATE IN FLEXURE
Illustrating Section F.2

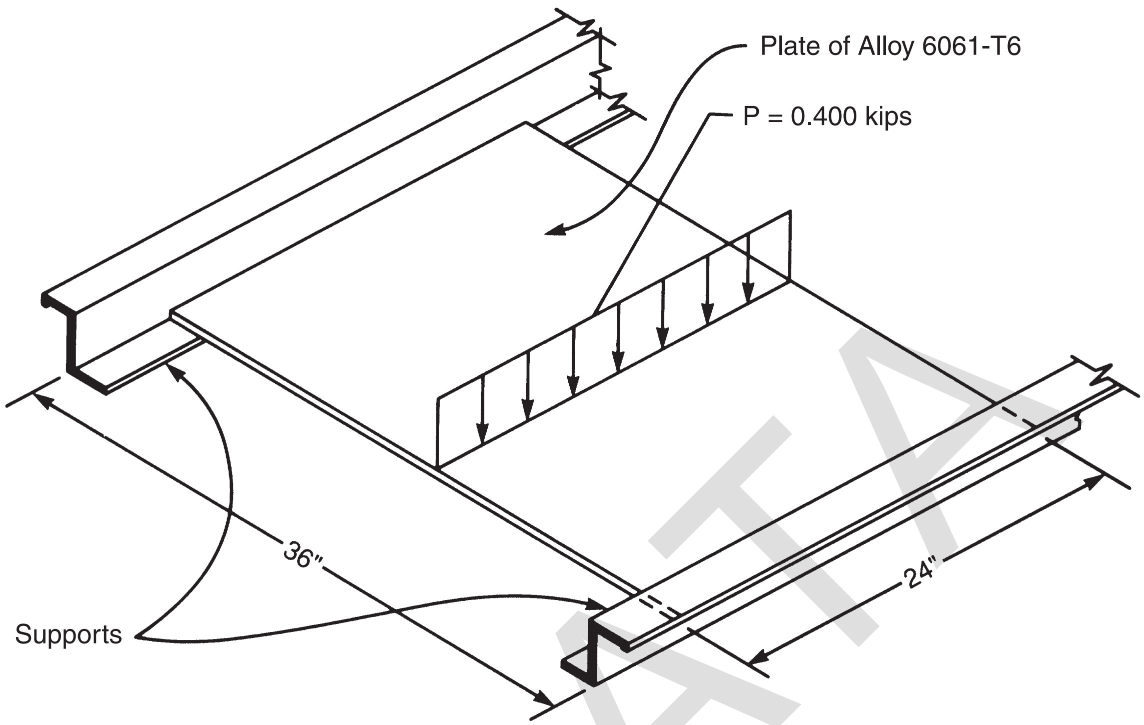

Figure 6

GIVEN:

1. Load 0.400 k, along a line at the center of a plate.
2. Plate: 24 in. wide, spanning 36 in.
3. Alloy: 6061-T6
4. Structure type: building

REQUIRED:

Minimum standard thickness to support the load safely without deflectingmorethan3/8in.

SOLUTION:

From Part VI, Beam Formulas Case 1, simply supported beam, concentrated load P at center

$M = PL/4 = (0.4)(36)/4 = 3.60$ in-k

For building-type structures, Section F.1 gives a safety factor of 1.95 for the rupture limit state and 1.65 for all other limit states.

The allowable yield moment M_{np}/Ω given in Section F.2 is the lesser of $1.5SF_{ty}/\Omega$ and ZF_{ty}/Ω; using F_{ty} = 35 ksi (see Table A.3.3), Ω = 1.65, and setting the allowable yield moment equal to the required moment:

$ZF_{ty}/\Omega = Z(35\ \text{k/in}^2)/1.65 = 3.60$ in-k

gives $Z = 0.170$ in^3.

and

$1.5SF_{ty}/\Omega = 1.5S(35\ \text{k/in}^2)/1.65 = 3.60$ in-k

gives $S = 0.113$ in^3.

The allowable moment for the limit state of rupture given in Section F.2 is $M_{nu}/\Omega = ZF_{tu}/k_t/\Omega$; using F_{tu} = 42 ksi and k_t = 1.0 (see Table A.3.3), Ω = 1.95, and setting the allowable moment equal to the required moment:

$ZF_{tu}/k_t/\Omega = Z(42\ \text{k/in}^2)/1.0/1.95 = 3.60$ in-k

gives $Z = 0.167$ in^3.

For a rectangle, the section modulus $S = bt^2/6$. Setting the section modulus equal to the required section modulus, and using $b = 24$ in. gives

$24t^2/6 = 0.113$ in^3, for which $t_1 = 0.168$ in.

For a rectangle, the plastic modulus $Z = bt^2/4$. Setting the plastic modulus equal to the required plastic modulus, and using $b = 24$ in. gives

$24t^2/4 = 0.170$ in^3, for which $t_1 = 0.168$ in.

Deflection

From Part VI Beam Formulas Case 1

Deflection = $PL^3/(48EI)$

A correction is required for plates because individual fi bers are restricted in the way they can change shape in the direction perpendicular to the stress. They can change in vertical dimension but not in horizontal dimension. The correction is:

$$\text{Deflection} = \Delta = \frac{PL^3(1-\nu^2)}{48EI}$$

where ν = Poisson's ratio, given in Table A.3.1 as 0.33.

From Part V Table 28 the moment of inertia for a rectangle is

$I = bt_2^3/12$

Section L.3 requires that bending deflections be determined using the compression modulus of elasticity from Table A.3.1, in which $E = 10{,}100$ ksi

Combining the equations for I and Δ,

$$t_2 = \sqrt[3]{\frac{PL^3(1-\nu^2)}{4bE\Delta}} = \sqrt[3]{\frac{(0.4)36^3(1-0.33^2)}{4(24)(10{,}100)(0.375)}} = 0.36 \text{ in.}$$

based on limiting deflection to 0.375 in..

Since $t_2 > t_1$ deflection controls; use 3/8 in. thick plate.

NOTES: The rails supporting the plate are assumed to have been checked structurally to see that they will safely support the load. They should be fastened to the plate at intervals to prevent spreading.

The loading and deflection limits in this example differ from those for Part VI Table 4-3.

From Part VI, Table 1-1, the buckling constants for the unwelded material are

$B_c = 32.6$ $B_p = 39.0$ $B_{br} = 52.0$
$D_c = 0.226$ $D_p = 0.297$ $D_{br} = 0.457$
$C_c = 95.9$ $C_p = 87.6$ $C_{br} = 75.8$

From Table B.4.3, $k_1 = 0.50$, $k_2 = 2.04$

For the portion of the cross section in the weld-affected zone, Table A.3.3 gives mechanical properties for 5456-H321 plate:

$E = 10{,}100$ ksi , $F_{tu} = 42$ ksi , $F_{ty} = 19$ ksi, $F_{cy} = 19$ ksi

From Part VI, Table 1-2, the buckling constants for the weld affected material are

$B_c = 21.6$ $B_p = 25.7$ $B_{br} = 34.1$
$D_c = 0.123$ $D_p = 0.158$ $D_{br} = 0.243$
$C_c = 117.7$ $C_p = 108$ $C_{br} = 93.6$

From Table B.4.3, $k_1 = 0.50$, $k_2 = 2.04$

Section F.1 establishes safety factors of 2.20 on tensile rupture and 1.85 on all other limit states for flexure of bridge-type structures.

a) Section F.2 addresses the yield and rupture limit states. The plastic neutral axis is located by equating the compressive yield force on the compression side of the plastic neutral axis and the tensile yield force on the tensile side of the plastic neutral axis. Conservatively use the compressive yield stress F_{cy} for both the compressive and tensile yield stresses.

Designating the distance from the bottom of the section to the plastic neutral axis as y, the compressive yield force is the sum of:

27.9(16 – 2.375)(1) = 380.1 k (compressive yield force in unwelded part of flange)

19(2.375)(1) = 45.1 k (compressive yield force in welded part of flange)

(19)(1)(0.375) = 7.1 k (compressive yield force in welded part of web)

27.9(50 – 2 – y)(0.375) = 502.2 k – 10.46y

The sum of the above is 934.5 k – 10.46y.

The tensile yield force is the sum of:

27.9(12 – 2.375)(1) = 268.5 k (tensile yield force in unwelded part of flange)

19(2.375)(1) = 45.1 k (tensile yield force in welded part of flange)

(19)(1)(0.375) = 7.1 k (tensile yield force in welded part of web)

27.9(y – 2)(0.375) = -20.9 k + 10.46y (tensile yield force in unwelded part of flange)

The sum is 299.8 k + 10.46y.

Setting the compressive yield force equal to the tensile yield force,

934.5 – 10.46y = 299.8 + 10.46y, y = 634.7/20.92 = 30.33”

The yield strength moment M_{np} is determined by summing the moments in each zone about the plastic neutral axis as tabulated below:

Table of Yield Strength Moments

Flange F or Web W	C or T	Welded W or Unwelded U	F_y k/in.²	*w* in.	*t* in.	*A=wt* in²	F_yA k	*ybar* in.	*ybar-y* in.	F_yA*(ybar-y)* in.-k
F	C	U	27.9	13.625	1	13.63	380.1	49.5	19.16	7,283
F	C	W	19	2.375	1	2.38	45.1	49.5	19.16	865
W	C	W	19	1	0.375	0.38	7.1	48.5	18.16	129
W	C	U	27.9	17.66	0.375	6.62	184.8	39.17	8.83	1,631
							617.2			
F	T	U	27.9	9.625	1	9.63	268.5	0.5	29.84	8,013
F	T	W	19	2.375	1	2.38	45.1	0.5	29.84	1,347
W	T	W	19	1	0.375	0.38	7.1	1.5	28.84	205
W	T	U	27.9	28.34	0.375	10.63	296.5	16.17	14.17	4,202
							617.3			
										M_{np} = 23,676

The allowable moment for the yield limit state is M_{np}/Ω = 23,676/1.85 = 12,800 in-k.

Since the weld affected area on the compression side of the neutral axis is same as on the tension side of the neutral axis, the location of the plastic neutral axis is the same if no part of the beam were weld affected, all of the beam were weld affected, or part of the beam is weld affected. Thus the plastic section modulus can be calculated as

Z = (16”)(1”)(49.5” – 30.33”) + (0.375”)(49” – 30.33”)^2/2 + (12”)(1”)(30.33” – 0.5”) + (0.375”)(30.33” – 1”)^2/2

$Z = 891 \text{ in}^3$

The yield limit state moment if no part of the section were weld-affected is

$M_{npo} = ZF_{cy}$ = (891 in^3)(27.9 k/in^2) = 24,870 in-k; M_{npo}/Ω = 24,870/1.85 = 13,440 in-k

The yield limit state moment if all of the section were weld-affected is

$M_{npw} = ZF_{cyw}$ = (891 in^3)(19 k/in^2) = 16,930 in-k; M_{npo}/Ω = 16,930/1.85 = 9,150 in-k

The rupture limit state moment is determined using the welded tensile strength of 42 ksi:

$M_{nuw} = ZF_{tu}$ = (891 in^3)(42 k/in^2) = 37,420 in-k; M_{npo}/Ω = 37,420/2.20 = 17,010 in-k

b) Section F.3 addresses local buckling.

Section B.5.4.1 addresses the flange. The slenderness ratio of the compression flange is

$b/t = (16 – 3/8)/2/1 = 7.8$

For the unwelded portion of the flange

$\lambda_1 = (B_p – F_{cy})/(5.0D_p) = (39.0 – 27.9)/(5.0(0.297)) = 7.5$

$$\lambda_2 = \frac{k_1 B_p}{5.0 D_p} = \frac{0.50(39.0)}{5.0(0.297)} = 13.1$$

$\lambda_1 = 7.5 < b/t = 7.8 < 13.1 = \lambda_1$, so
$F_{co} = B_p – 5.0D_p\, b/t = 39.0 – 5.0(0.297)(7.8) = 27.4$ ksi

F_{co} / Ω = (27.4 ksi)/1.85 = 14.8 ksi

For the welded portion of the flange

$\lambda_1 = (B_p – F_{cy})/(5.0D_p) = (25.7 – 19)/(5.0(0.158)) = 8.5$

$b/t = 7.8 < 8.5 = \lambda_1$, so
$F_{cw}/\Omega = F_{cyw}/\Omega$ = (19 ksi)/1.85 = 10.3 ksi

Section B.5.4 provides the strength of the compression flange as

$F_{cf} = F_{co}(1 – A_{wz}/A_g) + F_{cw} A_{wz}/A_g$

The gross area of the compression flange is

$A_g = 16(1) = 16 \text{ in}^2$

The weld-affected area of the compression flange is

$A_{wz} = 2.375 \text{ in}^2$

$F_{cf}/\Omega = [F_{co}(1 – A_{wz}/A_g) + F_{cw} A_{wz}/A_g] / \Omega$

F_{cf}/Ω = [14.8(1 – 2.375 /16) + 10.3 (2.375) /16]
= 14.1 ksi

Section B.5.5.1 addresses the web.

The slenderness ratio of the web is

$b/t = (50 – 2)/0.375 = 128$

$c_c = -22.9 + 1 = -21.9$

$c_o = 27.1 – 1 = 26.9$

$c_o/c_c = 26.9/-21.9 = -1.23$

$m = 1.3/(1 – c_o/c_c) = 1.3/(1 – (-1.23)) = 0.58$

For the unwelded portion of the web

$$\lambda_2 = \frac{k_1 B_{br}}{m D_{br}} = \frac{0.5(52.0)}{(0.58)(0.457)} = 98.1$$

$b/t = 128 > 98.1 = \lambda_2$, so

$$F_{bo} = \frac{k_2\sqrt{B_{br}E}}{mb/t} = \frac{2.04\sqrt{(52.0)(10{,}100)}}{(0.58)(128)} = 19.9 \text{ ksi}$$

F_{bo}/Ω = 19.9/1.85 = 10.8 ksi

For the welded portion of the web

$$\lambda_2 = \frac{k_1 B_{br}}{m D_{br}} = \frac{0.5(34.1)}{(0.58)(0.243)} = 121$$

$b/t = 128 > 121 = \lambda_2$, so

$$F_{bo} = \frac{k_2\sqrt{B_{br}E}}{mb/t} = \frac{2.04\sqrt{(34.1)(10{,}100)}}{(0.58)(128)} = 16.1 \text{ ksi}$$

F_{bw}/Ω = 16.1/1.85 = 8.7 ksi

Section B.5.5 provides the strength of the web in compression as

$F_b = F_{bo}(1 – A_{wzc}/A_{gc}) + F_{bw} A_{wzc}/A_{gc}$

The gross area of the web in compression is

$A_g = 0.375(22.9 – 1) = 8.21 \text{ in}^2$

The weld-affected area of the web in compression is

$A_{wz} = (1)(0.375) = 0.375 \text{ in}^2$

$F_b/\Omega = [F_{bo}(1 - A_{wzc}/A_{gc}) + F_{bw}A_{wzc}/A_{gc}]/\Omega$

$F_b/\Omega = [10.8(1 - 0.375/8.21) + 8.7(0.375)/8.21]$
$= 10.7$ ksi

Section F.3.1 provides the weighted average strength of the elements.

The moment of inertia of the flanges is

$I_f = (12)(1)^3/12 + (16)(1)^3/12 + (16)(1)(22.9 - 0.5)^2 + (12)(1)(27.1 - 0.5)^2 = 16{,}521 \text{ in}^4$

The moment of inertia of the web is

$I_w = (0.375)(48)^3/12 + (0.375)(48)(27.1 - 25)^2 = 3535 \text{ in}^4$

$M_{nlb} = F_{cf}I_f/c_{cf} + F_{cw}I_w/c_{cw}$

$M_{nc}/\Omega = (14.1)(16521)/(22.9 - 0.5) + (10.7)(3535)/(22.9 - 1) = 12{,}130$ in-k

c) Section F.4 addresses lateral-torsional buckling. To determine the slenderness ratio $\lambda = \dfrac{L_b}{r_y\sqrt{C_b}}$, Section F.1.1 allows the bending coefficient C_b to be conservatively taken as 1.

Since the compression flange is larger than the tension flange, Section F.4.2.2 does not apply. Section F.4.2.5 applies to any beam, so using it:

$$M_e = \frac{C_b\pi^2EI_y}{L_b^2}\left[U + \sqrt{U^2 + \frac{0.038JL_b^2}{I_y} + \frac{C_w}{I_y}}\right]$$

Conservatively assume $C_b = 1$.

$U = C_1g_0 - C_2\beta_x/2$. Section F.4.2.5 permits C_1 and C_2 to be taken as 0.5, so

$U = 0.5(27.1 + 7.85) - 0.5(17.9)/2 = 13.0$ in.

$$M_e = \frac{(1)\pi^2(10{,}100)(485.5)}{(120)^2} \times \left[13.0 + \sqrt{(13.0)^2 + \frac{0.038(10.2)(120)^2}{485.5} + \frac{243{,}400}{485.5}}\right]$$

$= 131{,}400$ in-k

$$\lambda = \pi\sqrt{\frac{ES_{xc}}{M_e}} = \pi\sqrt{\frac{(10100)(879)}{131400}} = 25.8$$

$$\frac{L_b}{r_y} = \frac{120}{3.69} = 32.5 < \lambda_2 = 1.2C_c = 1.2(103) = 124$$

For a beam with no portion weld-affected:

$\lambda = 25.8 < 95.9 = C_c$, so

$$M_{nmb} = M_{np}\left(1 - \frac{\lambda}{C_c}\right) + \frac{\pi^2E\lambda S_{xc}}{C_c^3}$$

$$= 24{,}970\left(1 - \frac{25.8}{95.9}\right) + \frac{\pi^2(10{,}100)(25.8)(879)}{95.9^3}$$

$= 20{,}740$ in-k

$F_b = M_{nmb}/S_x = (20{,}740 \text{ in-k})/(879 \text{ in}^3) = 23.6 \text{ k/in}^2$

For a beam entirely weld-affected:

$\lambda = 25.8 < 117.7 = C_c$, so

$$M_{nmb} = M_{np}\left(1 - \frac{\lambda}{C_c}\right) + \frac{\pi^2E\lambda S_{xc}}{C_c^3}$$

$$= 17.005\left(1 - \frac{25.8}{95.9}\right) + \frac{\pi^2(10{,}100)(25.8)(879}{95.9^3}$$

$= 14{,}940$ in-k

$F_b = M_{nmb}/S_x = (14{,}940 \text{ in-k})/(879 \text{ in}^3) = 17.0 \text{ k/in}^2$

Section F.4 provides the lateral-torsional buckling strength of longitudinally welded beams as

$M_n = M_{no}(1 - A_{wz}/A_f) + M_{nw}(A_{wz}/A_f)$

where

$A_{wz} = (1 + 0.375 + 1)(1) + (1)(0.375) = 2.75 \text{ in}^2$

$A_f = (16)(1) + (22.9/3 - 1)(0.375) = 18.5 \text{ in}^2$

$M_{nmb} = 20{,}740(1 - 2.75/18.5) + 14{,}940(2.75/18.5)$
$= 19{,}880$ in-k

$M_{nmb}/\Omega = 19{,}880/1.85 = 10{,}740$ in-k

The lateral-torsional buckling stress $F_b = M_{nmb}/S_c$

$F_b = (19{,}880 \text{ in-k}/879 \text{ in}^3) = 22.6 \text{ k/in}^2$

d) Section F.4.3 addresses interaction between local buckling and lateral-torsional buckling.

The flange's slenderness ratio is

$b/t = (16 - 0.375)/2/1 = 7.8$

The flange's elastic buckling stress given in Section B.5.6 is

$$F_{cr} = \frac{\pi^2E}{(5.0b/t)^2} = \frac{\pi^2(10{,}100)}{(5.0(7.8))^2} = 65.5 \text{ ksi} > 22.6 \text{ ksi}$$

Because the flange's elastic buckling stress is not less than the beam's lateral-torsional buckling stress, the beam's flexural capacity is not limited by the interaction between local buckling and lateral-torsional buckling.

The allowable moments are:

For yielding: M_{np} / Ω = 12,800 in-k
For rupture: M_{nu} / Ω = 17,010 in-k
For lateral-torsional buckling: M_n / Ω = 10,740 in-k
For local buckling: M_n / Ω = 12,130 in-k

The least of these is 10,740 in-k from lateral-torsional buckling.

Allowable moment based on fatigue per Appendix 3

Figure 3.1 detail 4 is similar to this example. Table 3.1 indicates that this detail is fatigue category B. Section 3.2 requires that for constant amplitude loading the applied stress range S_{ra} be less than the allowable stress range S_{rd}:

$$S_{ra} < S_{rd} = C_f N^{-1/m}$$

For category B, Table 3.2 gives C_f = 130 ksi and m = 4.84, so

$$S_{rd} = (130 \text{ ksi})/(500{,}000)^{1/4.84} = 8.6 \text{ ksi}$$

Assuming that there is no load reversal, the maximum stress equals the stress range. The section modulus corresponding to the weld on the tension flange is

$$S_w = 20{,}132/(27.1 - 1.0) = 771 \text{ in}^3$$

The tensile moment for fatigue M_f for the tensile stress range is

$$M_f = S_{rd} S_w = (8.6 \text{ k/in}^2)(771 \text{ in}^3) = 6630 \text{ in-k}$$

If variable amplitude loading occurred, an equivalent stress range would be calculated to compare to the allowable stress range. For example, if the loading were

100,000 cycles	9.5 ksi stress range
50,000 cycles	10.0 ksi stress range
350,000 cycles	7.1 ksi stress range
500,000 cycles	at various stress ranges

Section 3.3 provides the equivalent stress range S_{re} for variable amplitude loading:

$$S_{re} = [(100/500)9.5^{4.84} + (50/500)10.0^{4.84} + (350/500)7.1^{4.84}]^{1/4.84} = 8.2 \text{ ksi} < 8.6 \text{ ksi}$$

So this variable amplitude loading does not exceed the allowable stress range.

Selection of allowable moment

Comparing the allowable static (10,390 in-k) and fatigue (6630 in-k) moments, the allowable moment is 6630 in-k from fatigue.

NOTES: If the shape of the moment diagram is known the lateral-torsional buckling strength could be determined more precisely by using the bending coefficient C_b computed according to Section F.4.1.2.

Example 18
PIPE IN FLEXURE
Illustrating Sections F.2, F.3, and F.4

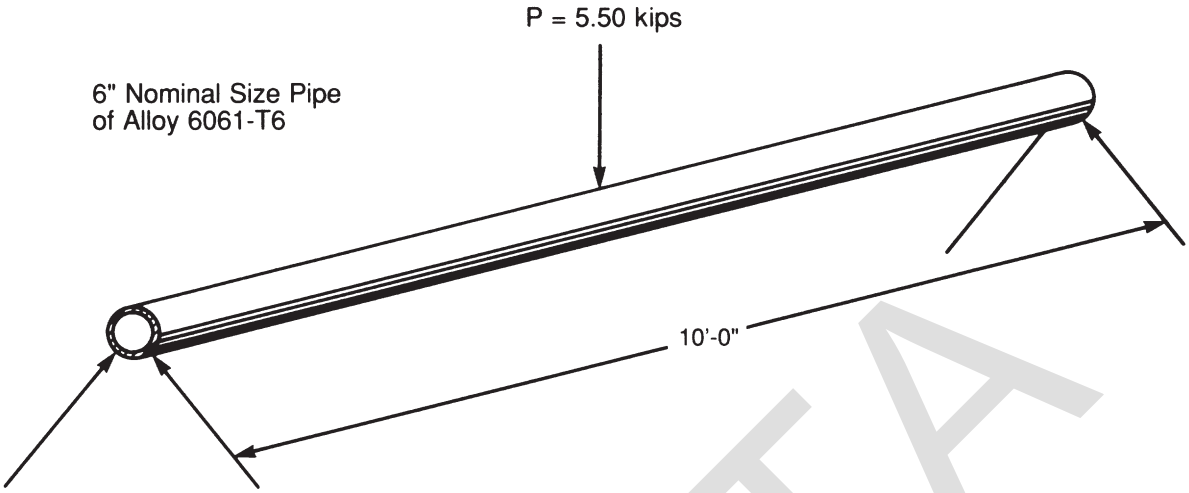

Figure 18

GIVEN:

1. Concentrated load of 5.5 k at mid-span.
2. Span: 10 ft, simply supported.
3. Alloy: 6061-T6.
4. Structure type: building

REQUIRED:

Is a 6 in. schedule 40 pipe adequate for the required load?

SOLUTION:

Section F.1 establishes safety factors of 1.95 on tensile rupture and 1.65 on all other limit states for fl exure of building-type structures. Allowable stresses for 6061-T6 given in Part VI Table 2-19 are used below.

Section F.2 addresses the limit states of yielding and rupture. Part V Table 22 shows, for a 6 in. schedule 40 pipe:

D = 6.625 in, t = 0.280 in., S = 8.50 in^3, Z = 11.3 in^3, I_y = 28.1 in^4, J = 56.2 in^4

For the limit state of yielding, the allowable moment is the lesser of

$$M_{np}/\Omega = 1.5SF_{ty}/\Omega = 1.5(8.50\text{ in}^3)(35\text{ k/in}^2)/1.65 = 270\text{ in-k}$$

$$M_{np}/\Omega = ZF_{ty}/\Omega = (11.3\text{ in}^3)(35\text{ k/in}^2)/1.65 = 239.7\text{ in-k}$$

The lesser of these is M_{np}/Ω = 239.7 in-k, and
M_{np} = (239.7 in-k)(1.65) = 395.5 in-k.

For the limit state of rupture, the allowable moment is

$$M_{nu}/\Omega = ZF_{tu}/k_t/\Omega = (11.3\text{ in}^3)(38\text{ k/in}^2)/1/1.95 = 220.2\text{ in-k.}$$

The allowable moment for local buckling determined using Section F.3.3 is based on Section B.5.5.4.

R_b/t = (6.625 – 0.280)/2/0.280 = 11.3 < 55.4 = λ_1, and
F_b/Ω = 39.3 – 2.7$(R_b/t)^{1/2}$ = 30.2 ksi

The allowable moment for local buckling is

$$M_{nlb}/\Omega = SF_b/\Omega = (8.50\text{ in}^3)(30.2\text{ k/in}^2) = 256.7\text{ in-k}$$

For closed shapes, the slenderness for lateral-torsional buckling using Section F.4.2.3 is

$$\lambda = 2.3\sqrt{\frac{L_b S_{xc}}{C_b\sqrt{I_y J}}} = 2.3\sqrt{\frac{(120)(8.50)}{(1)\sqrt{(28.1)(56.2)}}} =$$

= 11.7 < 66 = C_c, so

$$M_{nmb} = M_{np}\left(1-\frac{\lambda}{C_c}\right)+\frac{\pi^2 E\lambda S_{xc}}{C_c^3}$$

$$= 395.5\left(1-\frac{11.7}{66}\right)+\frac{\pi^2(10{,}100)(11.7)(8.50)}{66^3}$$

= 359.9 in-k

The allowable moment for lateral-torsional buckling is $M_{nmb}/\Omega = 359.9/1.65 = 218.1$ in-k

The allowable moment is the least of the allowable moments for yielding (239.7), rupture (220.2), local buckling (256.7), and lateral-torsional buckling (218.1), which is 218.1 in-k.

From Part VI Beam Formulas Case 1, a simply supported beam with a concentrated load *P* at center, the maximum moment is

$M = PL/4 = (5.5)(10)(12)/4 = 165$ in-k $<$ 218.1 in-k

The 6 in. schedule 40 pipe is therefore satisfactory.

Element Properties

Element	y	L	yL	y^2L	I
1	0.016	1.375	0.022	0.000	0.000
2	0.500	1.090	0.545	0.272	0.085
3	0.984	5.625	5.535	5.446	0.000
4	0.500	1.090	0.545	0.272	0.085
Totals		9.179	6.647	5.992	0.170

$c_t = \Sigma yL/\Sigma L = 6.647/9.179 = 0.724$ in., height of neutral axis

$I_x = [\Sigma(y^2L) - c_t^2\Sigma L + \Sigma I]t = [5.992 - (0.724)^2(9.179) + 0.170](0.032) = (1.349 \text{ in}^3)(0.032 \text{ in.})$

$I_x = 0.0432 \text{ in}^4$

$S_{\text{bot}} = I_x/c_t = (0.0432)/(0.724) = 0.0596 \text{ in}^3$

$S_{\text{top}} = I_x/(\text{height} - c_t) = (0.0432)/(1 - 0.724) = 0.1565 \text{ in}^3$

The moment of inertia of the flanges (elements 1 and 3) is

$I_f = [(1.375)(0.724-0.016)^2 + (5.625)(0.984-0.724)^2](0.032) = 0.0342 \text{ in}^4$

The moment of inertia of the webs (elements 2 and 4) is

$I_w = [2(1.090)(0.724 - 0.5)^2 + 2(0.085)]\ (0.032) = 0.0089 \text{ in}^4$

The plastic section modulus computed by finding the plastic neutral axis such that the area above this axis equals the area below is $Z = 0.0781 \text{ in}^3$.

Section F.1 states that the allowable moment is the least of the allowable moments for yielding, rupture, local buckling, and lateral-torsional buckling. Lateral-torsional buckling is unlikely to govern for this shape. Section F.1 also establishes safety factors of 1.95 on tensile rupture and 1.65 on all other limit states for flexure of building-type structures.

By Section F.2, the allowable moment for yielding is the least of

$1.5F_{cy}S_c/\Omega$

$1.5F_{ty}S_t/\Omega$

and ZF_{cy}/Ω

For the top flange in compression,

$1.5F_{cy}S_c/\Omega = 1.5(27)(0.1565)/1.65 = 3.84$ in-k

$1.5F_{ty}S_t/\Omega = 1.5(30)(\ 0.0596)/1.65 = 1.63$ in-k

and $ZF_{cy}/\Omega = (0.0781)(27)/1.65 = 1.28$ in-k,
so $M_{np}/\Omega = 1.28$ in-k.

For the bottom flange in compression,

$1.5F_{cy}S_c/\Omega = 1.5(27)(0.0596)/1.65 = 1.46$ in-k

$1.5F_{ty}S_t/\Omega = 1.5(30)(\ 0.1565)/1.65 = 4.27$ in-k

and $ZF_{cy}/\Omega = (0.0781)(27)/1.65 = 1.28$ in-k,
so $M_{np}/\Omega = 1.28$ in-k.

For the limit state of rupture, the allowable moment is

$M_{nu}/\Omega = ZF_{tu}/k_t/\Omega = (0.0781)(34)/1/1.95 = 1.36$ in-k.

The allowable moment for the limit state of local buckling is determined using Section F.3.1.

For the top flange in compression,

a) Element 3 is in uniform compression;
$b/t = \lambda = 5.625/0.032 = 175.8$.

By Section B.5.4.2, $\lambda_2 = 41.8$, so

$$F_c/\Omega = \frac{k_2\sqrt{B_p E}}{(1.6\,b/t)\Omega} = \frac{2.04\sqrt{(37.6)(10{,}100)}}{1.6(175.8)(1.65)} = 2.7 \text{ ksi}$$

b) Elements 2 and 4 are in flexural compression;
$b/t = \lambda = 1.09/0.032 = 34.1$.

By Section B.5.5.1:

$c_c = 0.724 - 1 = -0.276$ in., $c_o = 0.724$ in.;
since $c_o/c_c = 0.724/-0.276 = -2.62 < -1$,

$m = 1.3/(1 - c_o/c_c) = 1.3/(1 - (-2.62)) = 0.359$ and

$\lambda_1 = (B_{br} - 1.5F_{cy})/(mD_{br}) = (50.2 - 1.5(27))/0.359/0.433 = 62.4 > 34.1$,

so $F_b/\Omega = 1.5\ F_{cy}\ /\Omega = 1.5(27)\ /1.65 = 24.5$ ksi

$M_{nLB}/\Omega = (F_c/\Omega)\ I_f/c_{cf} + (F_b/\Omega)\ I_w/c_{cw}$

$M_{nLB}/\Omega = (2.7)\ (0.0342)/(1 - 0.724 - 0.032/2) + 24.5\ (0.0089)/(0.276 - 0.032) = 1.25$ in-k

For the bottom flange in compression,

a) Element 1 is in uniform compression;
$b/t = \lambda = 1.375/0.032 = 43.0$.

By Section B.5.4.2, $\lambda_2 = 41.8$, so

$$F_c/\Omega = \frac{k_2\sqrt{B_p E}}{(1.6\,b/t)\Omega} = \frac{2.04\sqrt{(37.6)(10{,}100)}}{1.6(43.0)(1.65)} = 11.1 \text{ ksi}$$

b) Elements 2 and 4 are in flexural compression;
$b/t = \lambda = 1.09/0.032 = 34.1$.

By Section B.5.5.1:

c_c = -0.724 in., c_o = 0.276 in.;
since c_o/c_c = 0.276/-0.724 = -0.381, and -1 < 0.381 < 1,

$m = 1.15 + c_o/2c_c$ = 1.15 + (-0.381/2) = 0.959 and

$\lambda_1 = (B_{br} - 1.5F_{cy})/(mD_{br})$ = (50.2 – 1.5(27))/0.959/0.433 = 23.4 < 34.1,

$\lambda_2 = k_1 B_{br}/(mD_{br})$ = 0.5(50.2)/0.959/0.433 = 60.4

so $F_b/\Omega = (B_{br} - mD_{br}\lambda)/\Omega$
= (50.2 – 0.959(0.433)(34.1))/1.65 = 21.8 ksi

$M_{nLB}/\Omega = (F_c I_f/\Omega)/c_{cf} + (F_b I_w/\Omega)/c_{cw}$

M_{nLB}/Ω = (11.1)(0.0342)/(0.724 – 0.032/2) + 21.8(0.0089)/(0.724 – 0.032) = 0.82 in-k

For the top flange in compression, the least of the allowable moments for yielding (1.28 in-k), rupture (1.36 in-k), and local buckling (1.25 in-k) is 1.25 in-k.

For the bottom flange in compression, the least of the allowable moments for yielding (1.28 in-k), rupture (1.36 in-k), and local buckling (0.82 in-k) is 0.82 in-k.

The above results can be converted to allowable moments per foot of width as follows:

M_{atc} = (1.25)(12 in./ft.)/(8 in./cycle)

= 1.87 k-in./ft-width (top in compression)

M_{abc} = (0.82)(12in./ft)/(8 in./cycle)

= 1.23 k-in./ft-width (bottom in compression)

2. Moment of inertia for deflection calculations

Refer to Section L.3

For element 1: $F_{cr} = \dfrac{\pi^2 E}{(1.6b/t)^2} = \dfrac{\pi^2(10{,}100)}{(1.6(43))^2}$

= 21.1 ksi > 11.1 ksi = f_a

so the width of element 1 is not reduced for deflection calculations.

For element 3: $F_{cr} = \dfrac{\pi^2 E}{(1.6b/t)^2} = \dfrac{\pi^2(10{,}100)}{(1.6(175.8))^2}$

= 1.3 ksi < 2.7 ksi = f_a

so the effective width of element 3 is

$b_e = b\,(F_{cr}/f_a)^{½}$
= 5.625 (1.3/2.7)^{½}
= 3.90 in.

Similarly, it can be seen that elements 2 and 4 are not reduced. A recalculation of the moment of inertia follows:

Element Properties

Element	y	L	L_{eff}	yL_{eff}	y^2L_{eff}	I_{eff}
1	0.016	1.375	1.375	0.022	0.000	0.000
2	0.500	1.090	1.090	0.545	0.272	0.085
3	0.984	5.625	3.90	3.838	3.78	0.000
4	0.500	1.090	1.090	0.545	0.272	0.085
Totals			7.455	4.95	4.32	0.170

$c_t = \sum (yL_{eff})/\sum L$ = 4.95/7.455

= 0.664 in., height of neutral axis

$I_x = [\sum(y^2L_{eff}) - c_t^2 \sum L_{eff} + \sum I_{eff}]t$
= [(4.32 – (0.664)^2(7.455) + 0.170)](0.032)
= (1.203 in^3)(0.032 in.)

= 0.0385 in^4, for deflection calculations when element 3 is at its allowable compressive stress.

3. Allowable reactions:

a. allowable interior reaction

Reference: Section J.9.1

Let the bearing length, N, be 2.0 in.

Consider element 2 (a web).

$P_c/\Omega = \dfrac{C_{wa}(N + C_{w1})}{\Omega C_{wb}}$

where $C_{wa} = t^2 \sin\theta\,(0.46F_{cy} + 0.02\sqrt{EF_{cy}}\,)$

where t = 0.032 in.

θ = 63.4°

F_{cy} = (0.9)(30) = 27 ksi

E = 10,100 ksi

so $C_{wa} = (0.032)^2 \sin 63.4°\,(0.46(27) + 0.02\sqrt{(10{,}100)(27)})$

C_{wa} = 0.0209 k

C_{w1} = 5.4 in.

$C_{wb} = C_{w3} + R_i\,(1 - \cos\theta)$
where
C_{w3} = 0.4 in.

R_i = 0.0625 in.

so

C_{wb} = 0.4 + 0.0625 (1 – cos 63.4°)

C_{wb} = 0.435 in.